# Frictional pressure drop of pure and contaminated bubble-electrolyte dispersions in electrochemical reactors

# H. VOGT

Fachbereich Verfahrens- und Umwelttechnik, Technische Fachhochschule Berlin, D-1000 Berlin 65, West Germany

Received 3 December 1982

An earlier equation for predicting the pressure drop in interelectrode gaps of gas-evolving electrodes and the connecting pipes is extended to contaminated electrolytes. Simplified design equations are given.

# Nomenclature

- $d_{\rm h}$  hydraulic diameter (m)
- $K_{\rm L}$  multiplier to Equation 1 (-)
- $K_{\rm v}$  numerical constant (-)
- n exponent to Equation 2 (-)
- p pressure drop of dispersion (kg m<sup>-1</sup> s<sup>-2</sup>)

 $p_{\rm L}$  single-phase liquid pressure drop (kg m<sup>-1</sup> s<sup>-2</sup>)

*Re* Reynolds number, 
$$Re = V_{\rm L} d_{\rm h} \rho_{\rm L} (S\eta_{\rm L})^{-1}$$

S cross-sectional flow area (m<sup>2</sup>)

$$V_{\rm G}, V_{\rm L}$$
 volumetric flow rate of gas and liquid,  
respectively (m<sup>3</sup> s<sup>-1</sup>)

- $\begin{array}{ll} x & \text{coordinate in flow direction (m)} \\ \eta & \text{dynamic viscosity of dispersion} \end{array}$
- $(\text{kg m}^{-1} \text{ s}^{-1})$  $\eta_{G}$  viscosity of gas (kg m<sup>-1</sup> s<sup>-1</sup>)
- $\eta_{\rm G}$  viscosity of gas (kg m<sup>-1</sup> s<sup>-1</sup>)  $\eta_{\rm L}$  viscosity of liquid (kg m<sup>-1</sup> s<sup>-1</sup>)
- $\eta_{\rm L}$  viscosity of liquid (kg m<sup>-1</sup> s
- $\rho_{\rm L}$  liquid density (kg m<sup>-3</sup>)
- $\phi$  volumetric gas fraction (-)

## 1. Introduction

In a previous communication a relationship for the prediction of the frictional pressure drop of bubble-electrolyte dispersions was derived [1]. The model described conditions in the gap between gas-evolving electrodes and the resulting equation is notable for its simplicity. The pressure drop is given by

$$\frac{\mathrm{d}p}{\mathrm{d}x} = K_{\mathrm{L}} \frac{\mathrm{d}p_{\mathrm{L}}}{\mathrm{d}x} \tag{1}$$

where  $dp_L$  expresses the frictional pressure drop as calculated for the liquid phase in the absence of gas under otherwise identical conditions over the flow length dx. The multiplier  $K_L$  is given by

$$K_{\rm L} = \frac{(1+\phi)^n}{1-\phi}$$
 (2)

where the gas fraction can be calculated from the volumetric flow rates of gas and liquid, respectively:

$$\phi = \frac{\dot{V}_{\mathbf{G}}}{\dot{V}_{\mathbf{G}} + \dot{V}_{\mathbf{L}}}.$$
(3)

The exponent *n* in Equation 2 is of value n = 1 for laminar flow and n = 0.25 for moderately turbulent flow,  $3000 < Re < 10^5$ .

It is the object of the present communication to generalize the multiplier to Equation 1 for reasons outlined below and, further, to simplify it to provide a useful and sufficiently accurate tool for electrochemical engineering purposes.

## 2. A generalized multiplier

The multiplier written in a more general form as compared to Equation 2 is given by [1]

$$K_{\mathbf{L}} = \frac{1}{1 - \phi} \left( \frac{\eta}{\eta_{\mathbf{L}}} \right)^n \tag{4}$$

where the ratio of the viscosities of the dispersion and the pure liquid was derived from the Einstein-Taylor equation for the effective viscosity of the dispersion

$$\frac{\eta}{\eta_{\rm L}} = 1 + 2.5\phi \frac{(\eta_{\rm G}/\eta_{\rm L}) + 0.4}{(\eta_{\rm G}/\eta_{\rm L}) + 1}$$
(5)

supposing that the gas-liquid interface of the

bubbles can freely move. This is the case for pure liquids as found in some laboratory set-ups and, moreover, in electrochemical reactors where the electrolyte is recycled and kept free from contaminants. With  $\eta_G/\eta_L \rightarrow 0$ , Equation 5 reduces to

$$\frac{\eta}{\eta_{\rm L}} = 1 + \phi \tag{6}$$

and Equation 4 coincides with Equation 2.

It has, however, been established by a number of investigators that bubbles in water and aqueous solutions accumulate surface-active contaminants at the gas-liquid interface. If their concentration is large enough the interface may become immovable [2, 3]. Such contaminants are often present in industrial electrolytes. The effect is the more pronounced the smaller the bubble size, presumably because the inertial forces are much smaller than the interfacial forces [4], and is particularly notable with bubble sizes as observed in gasevolving electrochemical reactors. The bubbles behave like rigid spheres, as though  $\eta_G/\eta_L \rightarrow \infty$ . In this case Equation 5 takes the form

$$\frac{\eta}{\eta_{\rm L}} = 1 + 2.5\phi \tag{7}$$

Hence, Equations 6 and 7 represent limiting cases for pure electrolytes with fully developed internal bubble circulation and for contaminated electrolytes where the gas bubbles act like solids with a rigid gas-electrolyte interface. Since comprehensive information on the interfacial properties of the gas-electrolyte system is not available, it is difficult to decide on the real behaviour of the dispersion in industrial reactors. In this case, we can only state that the real conditions encountered in the field will lie between those represented by Equations 6 and 7. Equation 2 should, therefore, be written in a more general form

$$K_{\rm L} = \frac{(1+K_{\rm v}\phi)^n}{1-\phi}, \ 1 \le K_{\rm v} \le 2.5$$
 (8)

where  $K_v = 1$  for pure electrolytes and  $K_v = 2.5$  for electrolytes sufficiently contaminated with surfactants.

#### 3. Modified multiplier

Since in most cases it would be questionable which



Fig. 1. The multiplier  $K_L$  against volumetric gas fraction  $\phi$  for laminar (n = 1) and moderately turbulent flow (n = 0.25), Equation 8.

value of  $K_v$  should be preferred it seems reasonable to propose a mean value for practical use. When taking  $K_v = 1.6$  the multiplier is too large as compared to the pure liquid and too small as compared to the contaminated electrolyte by less than  $\pm 16\%$  for values of  $\phi = 0.3$  and n = 1 (laminar flow). In turbulent flow, n = 0.25, the deviation is less than  $\pm 4\%$ , Fig. 1. The deviation increases slightly as the gas fraction  $\phi$  increases and vice versa. With regard to the accuracy of the whole procedure in predicting frictional pressure drop, the uncertainty may be considered acceptable. Therefore, a multiplier

$$K_{\rm L} = \frac{(1+1.6\phi)^n}{1-\phi} \tag{9}$$

instead of Equation 2 is proposed for cases where reliable information to predict the behaviour of

the gas-liquid interface is not available. At present, this will be so in the vast majority of cases.

It may finally be helpful to give Equation 9 in modified forms. In the case of *laminar* flow the multiplier to Equation 1 can be written as

$$K_{\rm L} = 1 + 2.6 \frac{\dot{V}_{\rm G}}{\dot{V}_{\rm L}} \tag{10}$$

resulting from Equation 9 together with Equation 3 for n = 1. The flow rates of gas and liquid are available from the operational data of the reactor. Note that  $\dot{V}_{\rm G}$  will vary strongly with the position in flow direction. Simple averaging procedures, however, will be serviceable.

In moderately *turbulent* flow (n = 0.25), Equation 9 can be simplified by writing

$$K_{\rm L} = 1 + 1.35 \, \frac{\dot{V}_{\rm G}}{\dot{V}_{\rm L}} \tag{11}$$

a form that follows for moderate values of  $\phi$  from a series extension. In the range of  $\phi < 0.4$  the deviation from Equation 8 does not exceed  $\pm 5\%$ , which shows that in turbulent flow the simple Equation 11 is not appreciably inferior to the exponential form of Equation 9.

In strongly turbulent flow (n = 0) with extremely large Reynolds numbers, a case which is rather rare in electrochemical engineering. Equations 2 and 9 may also be written as

$$K_{\rm L} = 1 + \frac{\dot{V}_{\rm G}}{\dot{V}_{\rm L}}.$$
 (12)

### Acknowledgement

The author is indebted to K. Roušar (Prague) for an important initiating remark.

#### References

- [1] H. Vogt, J. Appl. Electrochem. 12 (1982) 261.
- [2] U. Haas, H. Schmidt-Traub and H. Brauer, Chem.-Ing.-Tech. 44 (1972) 1060.
- [3] A. Weiner and S. W. Churchill, in 'Physicochemical Hydrodynamics' (edited by V. G. Levich Festschrift and D. B. Spalding) Advance Publ. Ltd., London (1977) p. 499.
- [4] J. H. Leonard and G. Houghton, Chem. Eng. Sci. 18 (1963) 133.